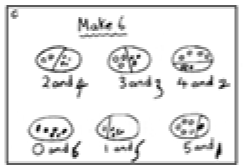
Starbeck Community Primary and Nursery School
Agreed Mathematics Calculation Methods

Addition

Guidance	Examples	
Pre: Playing with numbers, counting, ordering and sorting **Stage 1: Recording and developing mental pictures** • Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They experience practical calculation opportunities using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. They develop ways of recording calculations using pictures, etc.	**Pre:** counting in number rhymes and songs, counting objects during play such as cups on the table or fruit in the bowl. Ordering numbers – eg in the messy Maths tray, large tiles outside, numbered pebbles in the sand. Sorting practical objects and counting. One to one correspondence of numbers and objects. **Stage 1**   **One and one, two more**  **makes one, two three."** There are 3 people on the bus. Another person gets on. How many now?  	Initially recording of calculating should be done by **adults** to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process. <u>Whilst cameras are an excellent way of keeping a record of what children have done, they are not a substitute for the modelling of different ways of recording calculation procedures.</u>

Stage 2: Progression in the use of a number line

• To help children develop a sound understanding of numbers and to be able to use them confidently in calculation, there needs to progression in their use of number tracks and number lines

The labelled number line

• Children begin to use numbered lines to support their calculations counting on in ones.
• They select the biggest number first i.e. 8 and count on the smaller number in ones.

Stage 2

Children should experience a range of representations of number lines, such as the progression listed below.

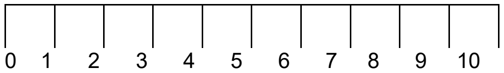
Physically moving or jumping on a large number line.

Filling in numbers on a blank number line.

Number track

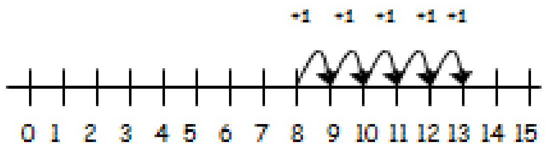
1	2	3	4	5	6	7	8	9	10

Number line, all numbers labelled



- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled

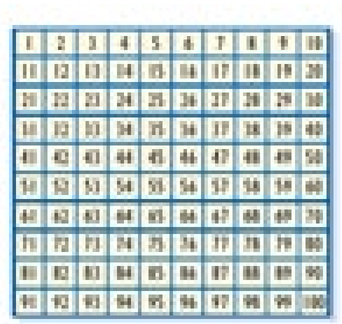
8 + 5 = 13



Additional 'number lines' - The bead string and hundred square

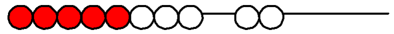
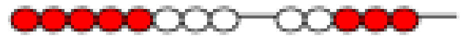
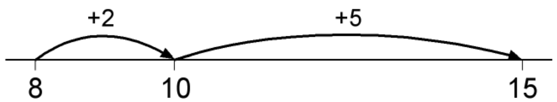
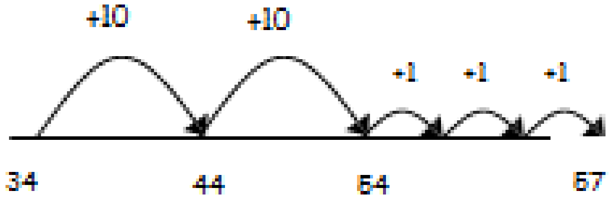
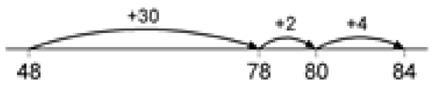
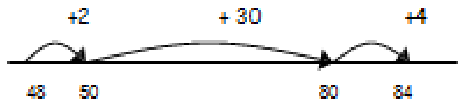
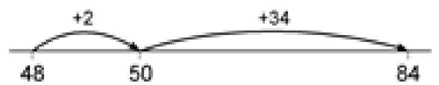
A hundred square is an efficient visual resource to support adding on in ones and tens and is an extension to the number track that children have experienced previously.

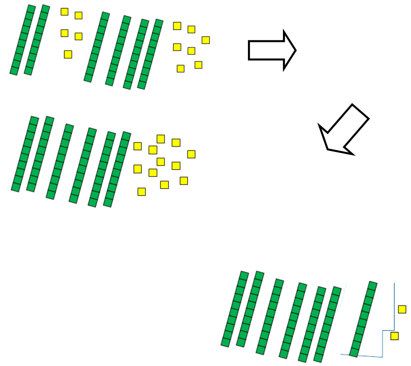
8 + 2 = 10

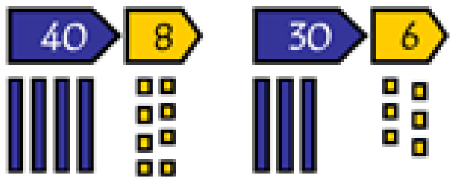


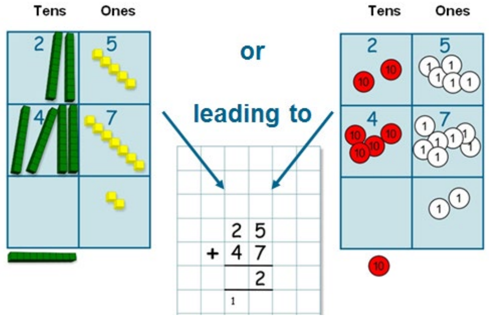
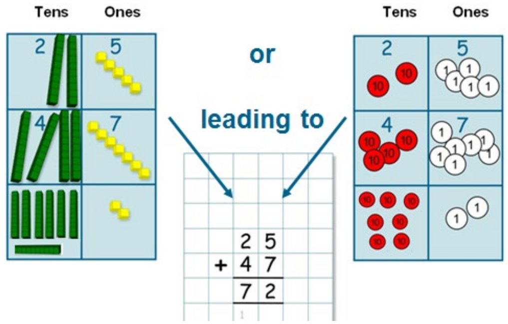
Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

Along with the number line, bead strings can be used to illustrate addition. Eight beads are counted out, then the two beads. Children count on from eight as they add the two beads e.g. starting at 8 they count 9 then 10 as they move the beads.

		Eight beads are counted out, then the five. Children count on from eight as they add the five e.g. starting at 8 they count 9, 10, 11, 12, 13.  8 + 5 = 13 
Stage 3: The empty number line as a representation of a mental strategy **NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.** • The mental methods that lead to column addition generally involve partitioning. • <u>Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.</u> • The empty number line helps to record the steps on the way to calculating the total.	Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. 8 + 7 = 15 Seven is partitioned into 2 and 5; 2 creating a number bond to 10 with the 8 and then the 5 is added to the 10.  First counting on in tens and ones. 34 + 23 = 57  This develops in efficiency, alongside children's confidence with place value.	Counting on in multiples of 10. 48 + 36 = 84  or  or  These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating

		with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**
Stage 4: Partitioning into tens and ones to lead to a formal written method • The next stage is to record mental methods using partitioning into tens and ones separately.  • Add the tens and then the ones to form partial sums and then add these partial sums. • Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods. • This method can be extended for TU + HTU and HTU + HTU and beyond; as well as cater for the addition of decimal numbers.	Stage 4 Children should use a range of practical apparatus (place value cards, straws, Dienes apparatus, place value counters) to complete TU + TU. They partition the number into tens and ones before adding the numbers together, finding the total. There should be progression through this selection of apparatus. Once using abstract representations teachers will start with straws, bundled into 10s and singularly. Children see 10 straws making one bundle and can be involved in bundling and unbundling. This then progresses to the use of Dienes (or similar) where 10s are clearly ten ones but cannot be separated in the same way. Once children are able to use these with understanding, they will progress to the use of place value cards and place value counters which are a further abstraction of the concept of number. Money should also be used (1ps, 10ps and £1) as place value equipment to help children develop their understanding of a range of representations. Progress through these manipulatives should be guided by understanding.	15 + 47  Children may make these jottings to support their calculation. 47 + 76 40 + 70 = 110 **or** 7 + 6 = 13 7 + 6 =13 40 + 70 = 110 110+ 13 = 123 110 + 13 = 123 **or** 47 + 70 = 117 117 + 6 = 123

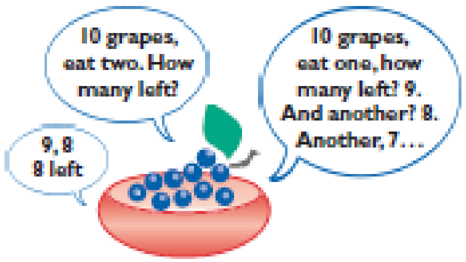
	48 + 36  40 + 30 = 70 8 + 6 = 14 70 + 14 = 84 Cuisenaire can also be used to support this step, especially when crossing the tens barrier with ones. When this occurs, children should use the term ‘exchange’ to describe converting ten ones into one ten.	
From Year 3 onwards **Stage 5 – Using Dienes/place value counters alongside columnar written method** • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column addition and when confident in explaining this, including exchanging when crossing the tens barrier with ones, they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations.	**It may be appropriate to teach children the process with numbers that they would be expected to calculate mentally or with jottings. This is to aid with the practicalities of the use of such equipment. However this should be the exception rather than the rule so children see a clear purpose for learning a new method for calculating.** In this example 25 + 47 =	Represented in place value columns and rows. Starting adding with the ‘least significant digit’ When the tens barriers is crossed in the ‘ones’ swapping then takes place.

• Children may learn more from experiencing the inefficiency of not starting with column with least significant value rather than being 'told' where to start.	  Whilst these images show the total existing alongside the two numbers being added, it may be more representative to 'drag' the manipulatives down to the totals box, leaving the written numbers as a reminder of what was originally there.	Because of the exchange we can know see that this ten belongs in the tens column and is carried there to be included in the total of that column. The tens are then added together 20 + 40 + 10 = 70, recorded as 7 in the tens column.
Stage 5: Compact column method • In this method, recording is reduced further. Carried digits are recorded, using the words 'carry ten' or 'carry one hundred' etc., according to the value of the digit. • Later the method is extended when adding more complex combinations such as three	**Stage 5** 258 + 87 = 345 (carried 1 1) 366 + 458 = 824 (carried 1 1) **Column addition remains efficient when used with larger whole numbers and once learned, is quick**	

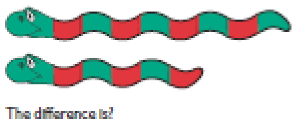
two-digit numbers, two three-digit numbers, and problems involving several numbers of different sizes.	**and reliable.** Th H T U 3674 + 2507 6181 1 1	

Starbeck Community Primary and Nursery School
Agreed Mathematics Calculation Methods

Subtraction

<table>
<tr><th>Guidance</th><th>Examples</th><th></th></tr>
<tr>
<td>
Pre: Playing with numbers, counting back.
Stage 1: Recording and developing mental pictures
□Children are encouraged to develop a mental picture of the calculation in their heads. They experience practical activities using a variety of equipment and develop ways to record their findings including models and pictures.
Find the difference is introduced through practical situations and images.
</td>
<td>
Pre: Counting in number rhymes and songs, counting back in songs, Counting back in a range or practical activities.
Stage 1


There are four children in the home corner. One leaves. How many are left?

</td>
<td>Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process, using their own informal methods of recording.</td>
</tr>
<tr>
<td> Stage 2: Progression in the use of a number line
• Finding out how many items are left </td>
<td> Stage 2
Children should experience a range of representations </td>
<td> Additional ‘number lines’ - The bead string and hundred square
A hundred square is an efficient visual resource to </td>
</tr>
</table>

after some have been 'taken away' is initially supported with a number track followed by labelled, unlabelled and finally empty number lines, as with addition.

The labelled number line

• The labelled number line, linked with previous learning experiences, is used to support calculations where the result is less objects (i.e. taking away) by counting back.

Difference between

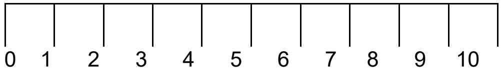
• The number line should also be used to make comparisons between numbers, to show that 6 – 3 means the '**difference in value** between 6 and 3' or the 'difference between3 and 6' and how many jumps they are apart.

of number lines, such as the progression listed below.

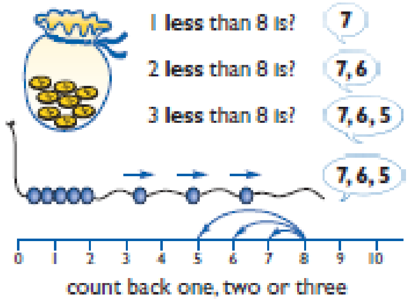
Number track

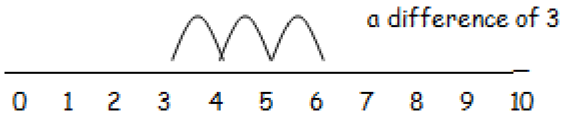
1	2	3	4	5	6	7	8	9	10

Number line, all numbers labelled

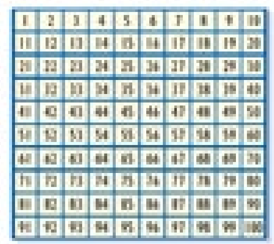


- Number line, 5s and 10s labelled
- Number line, 10s labelled
- Number lines, marked but unlabelled



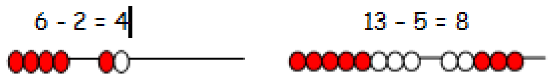


support counting on and back in ones and tens and is an extension of the number track which children have experienced previously



Different orientations of the 100 square help children transfer their skills and understanding between similar representations.

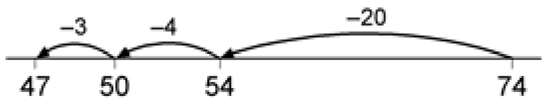
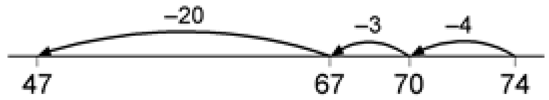
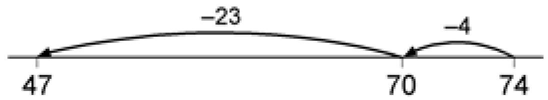
- Bead strings can be used to illustrate subtraction. 6 beads are counted and then the 2 beads taken away to leave 4.

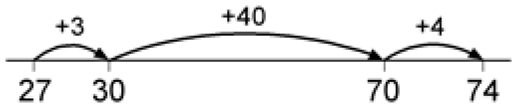
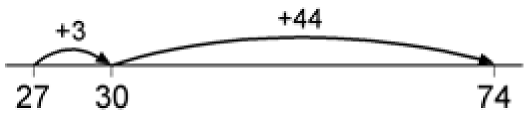
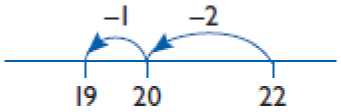
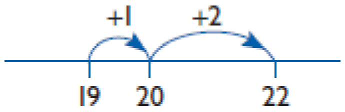


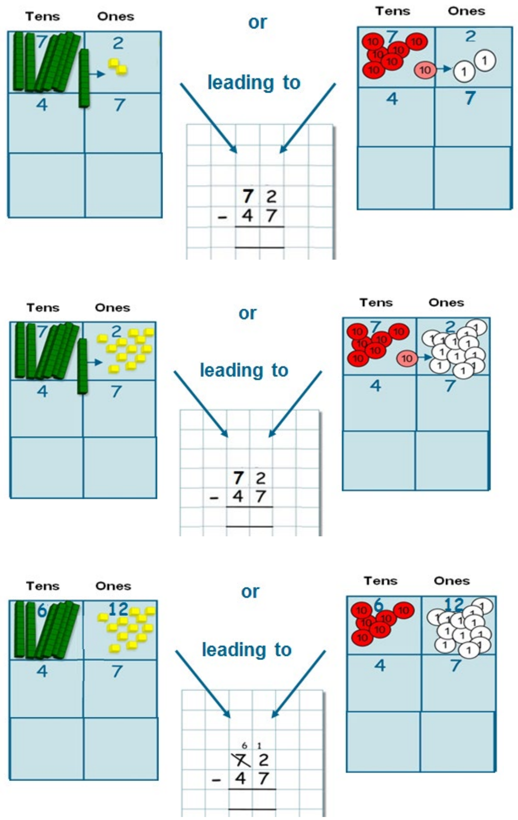
Stage 3: The empty number line as a representation of a mental

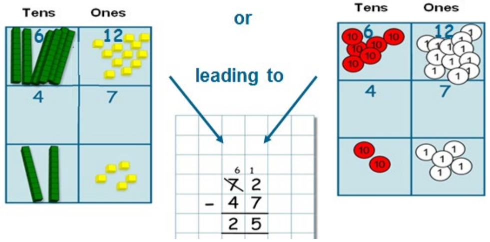
Steps in subtraction can be recorded on a number line.

The steps may be recorded in a different order:

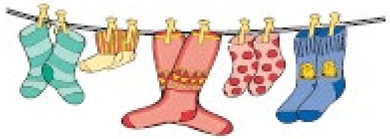
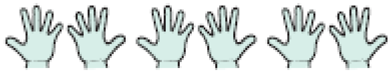
strategy **NB It is important to note that the empty number line is intended to be a representation of a mental method, not a written algorithm (method). Therefore the order and size (physical and numerical) of the jumps should be expected to vary from one calculation to the next.** **Finding an answer by COUNTING BACK** • **Counting back** is a useful strategy when the context of the problem results in there being less e.g. Bill has 15 sweets and gives 7 to his friend Jack, how many does he have left? As in addition, children need to be able to partition numbers e.g. the 7 is partitioned into 5 and 2 to enable counting back to 10. • The empty number line helps to record or explain the steps in mental subtraction. • A calculation like 74 – 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is a useful way of modelling processes such as	The steps often bridge through a multiple of 10. 15 – 7 = 8 The seven is partitioned into 5 (to allow count back to 10) and two.  74 – 27 = 47 worked by counting back: 	 or combined  These examples show how children should be taught to use jumps of different sizes, and completed in an order that is most helpful depending on the numbers they are calculating with. **This reinforces that this is a visual representation of a mental method and not a written alogrithm.**

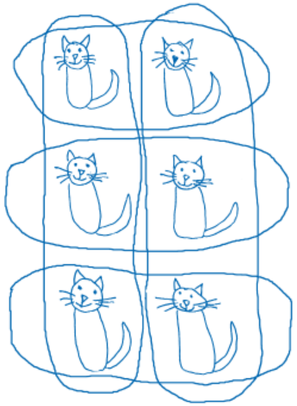
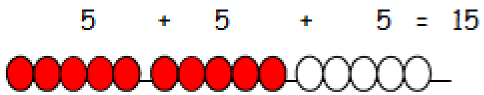
bridging through a multiple of ten.		
Using an empty number line **Finding an answer by COUNTING ON** • The steps can also be recorded by **counting on** from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47 (shopkeeper's method). This is a useful method when the context asks for comparisons e.g. how much longer, how much smaller; for example: Jill has knitted 27cm of her scarf, Alex has knitted 74cm. How much longer is Alex's scarf? After practice of both, examples like this will illustrate how children might choose when it is appropriate to count on or back. This also helps to reinforce addition and subtraction as inverses and the links between known number facts.	74 - 27 =  The 'jumps' should be added, either mentally or with jottings according to confidence, beginning with the largest number e.g. 40 + 4 + 3. or  22 – 3  22 – 19 = 3  	

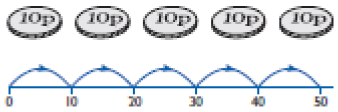
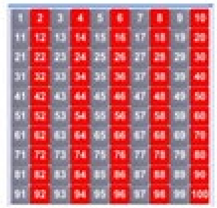
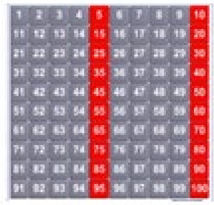
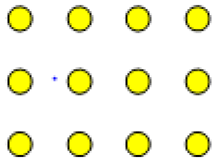
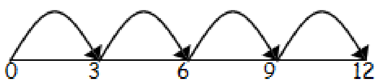
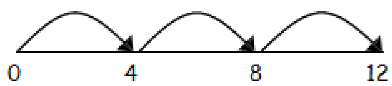
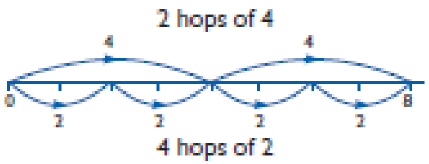
From Year 3 onwards **Stage 4: Making the link between the practical and columnar subtraction** • To ensure the statutory final written method is grounded in understanding, this stage connects the practical equipment to the formal written method using a similar and transferrable layout. • Children first experience the practical version of column subtraction and when confident in explaining this, including exchanging when ‘not having enough to subtract from’, they record the written method alongside. • Ideally children will experience this stage with a variety of practical equipment to make sure their understanding is embedded and transferrable between representations. • Children may learn more from experiencing the inefficiency of not starting with column with least significant value than being ‘told’ where to start.	72 - 47  	Whilst the images here show the total existing alongside the original number, it is suggested that the 47 would be ‘removed’ from the original set, before ‘dragging’ what is left down to the totals box. This would more closely represent the written algorithm.

		
Stage 5: Compact method • Finally children complete the compact columnar subtraction as the most efficient form. • Once children are confident with HTU – HTU, this should be extended to four digit subtract four digit calculations.	563 – 246 = 317 5 1 563 246 317 932 - 457 becomes 8 12 1 9 3 2 - 4 5 7 4 7 5 Answer: 475	When borrowing in column subtractions, numbers should be recorded at the top and then crossed out.

Multiplication

Guidance	**Examples**	
Stage 1: Recording and developing mental images • Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. • They will experience practical calculation opportunities involving equal sets or groups using a wide variety of equipment, e.g. small world play, role play, counters, cubes etc. • They develop ways of recording calculations using pictures, etc. • They will see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning answering questions such as; 'How	**Stage 1**  2 + 2 + 2 + 2 + 2 = 10  5 + 5 + 5 + 5 + 5 + 5 = 30 5 × 6 = 30  2 groups of 3 are 6 (3 + 3) 3 groups of 2 are 6 (2 + 2 + 2)	Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.

many eggs would we need to fill the egg box? How do you know?' • Children will use repeated addition to carry out multiplication supported by the use of counters/cubes.	 4 lots of 3 are 12 3 lots of 4 are 12  Children should use pictorial representations and may use rings to show e.g. 3 groups of 2 and 2 groups of 3 introducing the commutative law of multiplication	
Stage 2: The bead string, number line and hundred square • Children continue to use repeated addition to carry out multiplication tasks and represent their counting on a bead string or a number line. • On a bead string, children count out three lots of 5 then count the beads altogether. • On a number line. Children	**Stage 2** 3 lots of 5 	Children begin pattern work on a 100 square to help them begin to recognise multiples and rules of divisibility.

count on in groups of 5. • These models illustrate how multiplication relates to repeated addition.	 10p + 10p + 10p + 10p + 10p = 50p 10p × 5 = 50p 5 hops of 10	 Multiples of 2  Multiples of 5 Children regularly sing songs, chant and play games to reinforce times tables facts and their associated patterns.
Stage 3: Arrays It is important to be able to visualise multiplication as a rectangular array. This helps children develop their understanding of the commutative law i.e. 3 x 4 = 4 x 3 The rectangular array allows the total to be found by repeated addition and the link can be made to the 'x' sign and associated vocabulary of 'lots of' 'groups of' etc.	**Stage 3** 3 lots of 4 3 X 4  4 lots of 3 4 x 3  4 lots of 3 = 12  3 lots of 4 =12	The relationship between the array and the number line showing both repeated additions should be demonstrated alongside each other  For more direct comparison, this could then be demonstrated on a single number line as appropriate.

Stage 4: Short multiplication for up to TU x 12

- The recording is reduced further, with the carried digits recorded either below the line or at the top of the next column.
- This method is appropriate for multiplying two and three digit numbers by numbers up to 12, which relies on children have recall of their times table facts up to 12.

342 × 7 becomes

	3	4	2
×			7
2	3	9	4
	2	1	

Answer: 2394

Digits are carried below the line

Long multiplication

- Each digit continues to be multiplied by each digit, but the totals are recorded in a more compact form, using 'carrying'
- Children's understanding of place value is vital so they recognise when they are multiplying tens, hundreds etc. they record their answer in the correct columns.
- Children should be able to explain each step of the process, initially relating it back to previous methods and experiences. They should be

124 × 26 becomes

	1	2	
	1	2	4
×		2	6
	7	4	4
2	4	8	0
3	2	2	4
1	1		

Answer: 3224

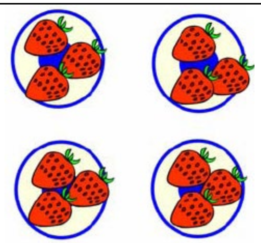
6 x 4 = 24 so record the 4 in the units and carry the 20 (2) into the tens 6 x 20 = 120 + (the carried) 20 = 140 so record the 40 in the tens and carry the 100 (1) into the hundreds column. 6 x 100 = 600 + (the carried) 100 = 700. Record as 7 in the hundreds.

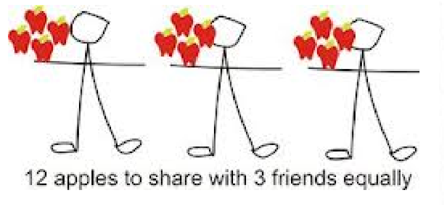
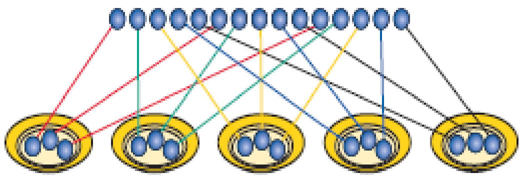
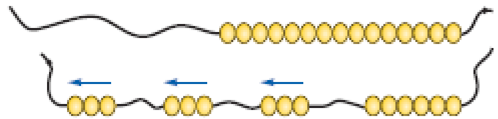
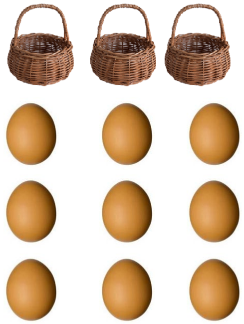
20 x 4 = 80 so record this on a new answer row in the correct columns. 20 x 20 = 400. Record the 4 in the hundreds column. 20 x 100 = 2000 so record this appropriately.

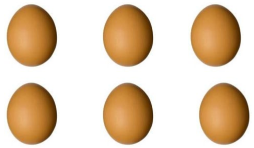
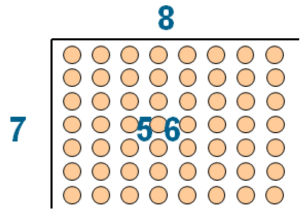
Use column addition to add the two totals together, resulting in 3224.

able to articulate the different stages of this calculation with the true values of the digits they are dealing with.		

Division

Stage 1: Recording and developing mental images • Children are encouraged, through practical experiences, to develop physical and mental images. • They make recordings of their work as they solve problems where they want to make equal groups of items or sharing objects out equally.		Initially recording of calculating should be done by adults to model what children have done in pictures, symbols, numbers and words. Over time there should be an expectation that children will also become involved in the recording process.

		
Sharing and Grouping • They solve sharing problems by using a 'one for you, one for me' strategy until all of the items have been given out. • Children should find the answer by counting how many eggs **1 basket** has got. • They solve grouping problems by creating groups of the given number. • Children should find the answer by counting out the eggs and finding out how many **groups of 3** there are. • They will begin to use their own jottings to record division	15 eggs are shared between 5 baskets. How many in each basket? First egg to the first basket, 2nd egg to the second etc  There are 15 eggs. How many baskets can we make with 3 eggs in? 	
Stage 2: Bead strings, number lines simple multiples • Using a bead string, children can represent division problems • They count on in equal steps based on adding multiples up to the number to be divided. • When packing eggs into baskets of	15 eggs are placed in baskets, with 3 in each basket. How many baskets are needed?  Counting on a labelled and then blank number	 3 eggs once 3 eggs twice 3 eggs three times 3 eggs four times 3 eggs five times

three they count in threes - **grouping** • If the problem requires 15 eggs to be **shared** between 3 baskets, the multiple of three is obtained each time all three baskets have received an egg.	lines. 15 ÷ 3 = 5 	
• **Stage 3: Arrays for division** Children construct arrays by grouping the dividend into groups of the divisor. The number of groups made is recorded as the quotient.	The use of arrays help to reinforce the link between multiplication and division 	Divided (56) ÷ divisor (7) = Quotient (8)
Stage 4: Short division Once children have developed a sound understanding of division, using the manipulatives 'formal written methods' of short and then long division. For calculations where numbers with up to 4 digits are divided by a single digit number, children are expected to use short division.	**Stage 4** Short division 432 ÷ 5 becomes 8 6 5 | 4 3 ³2 Answer: 86 remainder 2 With short division, children are expected to 'internalise' the working from above	By the time children are ready for long division, manipulatives may not aid calculating, however they may aid the understanding of the process of long division. The steps followed can be described as those followed when using PVCs to divide e.g. How many groups of 15 hundreds can we make?

Stage 5: Long division
For calculations where numbers of up to 4 digits are divided by a two digit number, children are expected to use long division.
	Stage 5
	Long division
	432 ÷ 15 becomes
	      2 8
	1 5 | 4 3 2
	      3 0 0
	      1 3 2
	      1 2 0
	        1 2
	Answer: 28 remainder 12
		None so we exchange the 4 hundreds for 40 tens.
		How many groups of 15 tens can we make? 2, equivalent to 300. We record the 2 and subtract the 300 that we have ‘organised’ from the dividend.
		We are now left with 132 ‘ones’. How many groups of 15 can we make with these? 8 and we have 12 left over.